FINITE-DIFFERENCE METHOD FOR THE ARBITRARY CROSS-SECTION WAVEGUIDE PROBLEM USING THE BEST-FIT BOUNDARY APPROXIMATION

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ABSTRACT

The finite-difference method is a numerical technique which accuracy strongly depends on the precision of the reconstruction of the boundary of a waveguide. Usually the figure is reconstructed either by an outside or an inside approximation, or by using a much more complex method of unequal-arm finite-difference operator. This paper describes a computer program which maintains the simplicity of the usual finite-difference method and which uses the best-fit approximation to the cross-section of a waveguide. As a result a better precision is obtained and smaller computer time is used in the solution of the dominant mode of the hollow homogeneous waveguide problem. Computed values of the cutoff frequencies for several waveguides are presented.

Introduction

The finite-difference method, when applied to the waveguide problem, transforms the Helmholtz equation of a continuous potential into a matrix equation of a discrete potential. The eigenvalues of this matrix are related to the cutoff wavelenths (k_c) of the waveguide. Depending on the boundary conditions used, we can get the TM or TE modes of the waveguide. The discretization can be achieved using an equal-arm[1] or unequal-arm[2] operator. The first one is very straightforward and easy to be implemented on the computer, while the second one requires more logic to accomodate the information about the mesh size of the operator near the curved boundaries and more computer storage area as a consequence. However the last one gives more accurate results because it reproduces better the boundary of the cross-section of a given waveguide. In this work an imporved equal-arm operator method is used and the accuracy of the obtained results is comparable with the ones of the unequal-arm operator method.

Description of the Method

At the interior points or when the boundaries are rectangular, both methods utilize the same mesh size (h). The difference appears near the curved boundaries. In the equal-arm operator method, described in[1], these boundaries are approximated from inside or from out-side. The unequal-arm operator method, described in[2], changes the size of the mesh so that it fits perfectly the boundary from inside. Because the aim of this work is to compute the fundamental mode only, it was judged that the equal--arm method could be improved without going to the complexities of the unequal-arm method.

The program accepts the analytical equation of the boundary of the waveguide. Using a given mesh size the program covers the cross-section of the waveguide with a square mesh. The problem of the boundary points is solved by the following logic.

The program detects the transition from the interior to the exterior points. Starting from an interior point on the transition, it computes two distances along the mesh line. The first one (d_1) is the distante from the interior point to the real boundary given by its analytical expression. The second one (d_2) is the distance from its exterior neighbor point to the real boundary. If d_2 is less than d_1, the corresponding exterior point becomes a part of the boundary. If not, it remains as an exterior point. In this way, even though the reconstruction of the boundary is not perfect, it can be considered as the best-fit attained using an equal-arm operator.

The solution of the matrix generated by the finite-difference scheme is iterative one and it starts with some assumed value for the discrete potentials and the eigenvalue. Usually[1] the program starts with the values of potentials equal to 1 and using an estimated eigenvalue computed by applying a matrix inversion method to a matrix generated by a rougher mesh size. This program starts with the eigenvalue equal to 1 for desired mesh size needing none of the matrix inversion methods.

In order to speed up the process, an optimal acceleration factor (ω_{opt}) is computed by the program.

$$\omega_{opt} = \frac{2}{1 + \sqrt{1 - \eta}},$$

where η is an estimate of the spectral radius of the matrix. It is the quotient of the norms of the two consecutive interactions.

Using this optimal acceleration factor, several iteractions are performed and the potentials are computed. The Rayleigh quotient give us an estimate of the eigenvalue. If two successive estimates of the eigenvalue differ by less than a pre-fixed precision the process is stopped. If not, it proceeds in order to satisfy this requirement.

Numerical Results

The program was run for several figures and the results were compared with[1]. The comparison for a circle appears in Table I.

TABLE I

$k_c d$ FOR TE_{11} AND TM_{01} MODES IN A CIRCULAR WAVEGUIDE

	TE_{11}				TM_{11}					
	FROM [1]		COMPUTED		FROM [1]				COMPUTED	
					$k_c d$		ERROR(%)			
h/d	$k_c d$	ERROR(%)	$k_c d$	ERROR(%)	OUTSIDE	INSIDE	OUTSIDE	INSIDE	$k_c d$	ERROR(%)
1/8	3 4389	-6 6	3 6797	0.06	4 5313	5 6665	-5 8	17.7	5 0063	4 09
1/16	3.5269	-4 2	3.6984	0.45	4.6648	5 2471	-3.0	9.1	4.8691	1.24
1/32	3 5978	-2 3	3 6905	0 23	4 7251	4 9996	-1.7	3 9	4 8548	0.94
1/64	3.6329	-1.3	3.6830	0.03	4.7622	4.8394	-1.0	1.9	4.8225	0.27
THEORETICAL	$k_c d$ = 3 6820				$k_c d$ = 4 8096					

d - Diameter of the circle.

It can be seen that the results of this work are better than that of[1] even for larger mesh sizes. This is due to better reconstruction of the boundary in our program. The same kind of behavior was observed for an ellipse. As far as a rectangle or a square is concerned, our results and theirs are very much similar for the TM-mode. This is explained by very good reconstruction of the boundary of these waveguides in both approaches, ours and theirs.

The comparison with[2] for the TE_{11} and TM_{01} modes of a circular waveguide appears in Table II. The normalized characteristic impedance of waveguide is defined on the power-voltage basis for the TM modes, and on the power-current basis for the TE modes.

TABLE II

$k_c d$ FOR TE_{11} AND TM_{01} MODES IN A CIRCULAR WAVEGUIDE

	TE_{11}				TM_{01}			
	FROM [2]		COMPUTED		FROM [2]		COMPUTED	
h/d	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)
1/10	1 8497 0.46	- -	1.8205 -1 12	1.1144 18.74	2.4040 -0 03	- -	2 3896 -0 63	0.1939 -5.04
1/20	1 8444 0 174	- -	1 8408 -0 02	1 0045 7.03	2 4035 -0 05	- -	2 4127 0.33	0 2015 -1 32
1/40	1 8423 0 059	- -	1 8396 -0.09	0.9498 1.21	2 4040 -0 03	- -	2 4091 0.18	0 2035 -0.34
.		0.9210 -1.8				0 2046 0.19		
THEORETICAL	1 8412	0 9385			2 4048	0.2042		

d - Radius of the circle.

Our results are somewhat worse than than theirs for larger mesh sizes, becoming equivalent to theirs when the mesh sizes decrease.

Even though it was not our goal to study the higher order modes, this was done for the third TE (TE_{01}) and TM (TM_{21}) modes of a circle imposing TE and TM boundary conditions over a quarter of a circle. The results are shown in Table II. The general behavior is similar to that of the fundamental modes.

TABLE III

3rd TE AND TM MODES IN A CIRCULAR WAVEGUIDE

	TE_{01}				TM_{21}			
	FROM [2]		COMPUTED		FROM [2]		COMPUTED	
h/d	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)	$k_c d$ ERROR(%)	IMPEDANCE Z_∞ ERROR(%)
1/10	3 8691 0 98	- -	2 9898 -21 97	- -	5 1025 -0 64	- -	5 0942 -0 81	.1570 -20 75
1/20	3 8467 0 39	- -	3 0783 -19.66	- -	5 1271 -0 17	- -	5 1557 0 39	1845 - 6.87
1/32	- -	- -	3.8339 + 0.06	- -	- -	- -	5.1511 0 30	.1924 - 2 88
1/40	3.8386 0.18	- -	3.8301 - 0.04	- -	5 1334 - 0 04	- -	5 1443 0 17	1950 - 1 56
						0 1975 - 0.30		
THEORETICAL	3 8317	∞			5.1356	0 1981		

d - Radius of the circle.

One figure which represents certain difficulty in its computation by the finite-difference method is an hexagon. The difficulty arises from the fact that it contains sharp angles and there is a $\sqrt{3}$ slope of its sides, making more difficult a precise reconstruction of the boundaries of the figure. The results for the TE and TM fundamental modes appear in the Table IV.

TABLE IV

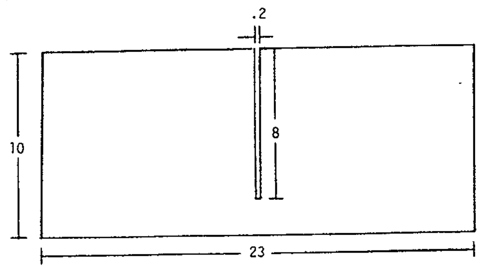
Z_∞ AND $k_c d$ FOR FUNDAMENTAL TE AND TM MODES

IN AN HEXAGONAL WAVEGUIDE

	TE		TM		
h/d	$k_c d$	IMPEDANCE (Z_∞)	$k_c d$	ERROR (%)	IMPEDANCE (Z_∞)
1/20	3 9622	1.1964	5.4037	0.98	0.1896
1/40	3.8978	1.0447	5.3552	0.08	0.2002
1/80	3.9011	1.0008	5.3451	-0.11	0.2031
THEORETICAL			5.35088		

d - Diameter of the circunscript circle.

It was also examined a ridge waveguide shown on Figure 1.



All dimensions in mm.

FIGURE 1 - RIDGE WAVEGUIDE

Its ($k_c d$) is find to be 1.9378 while its impedance is 0.5932.

Conclusion

The computer program presented in this paper showed very good performance as far as the cutoff wave length and the impedance are concerned. It required less than 8 min of computer time to evaluate about 3000 points for getting ($k_c d$) with 10^{-4} precision. If this requirement is relaxed the computing time decreases rapidly.

The program was run on the Burroughs B-6800 machine and it required up to 10K of storage for most of the figures described in this work and up to 290K for very large figures like the ridge.

References

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2 - BEAUBIEN, M.J. and A. WEXLER "Unequal-Arm Finite-Difference Operators in the Positive-Definite Successive Overrelation (PDSOR) Algorithm", IEE Transactions Microwave Theory and Technique, vol. MTT-11, no. 12, Dec. 1970, pp. 1132-1149.

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